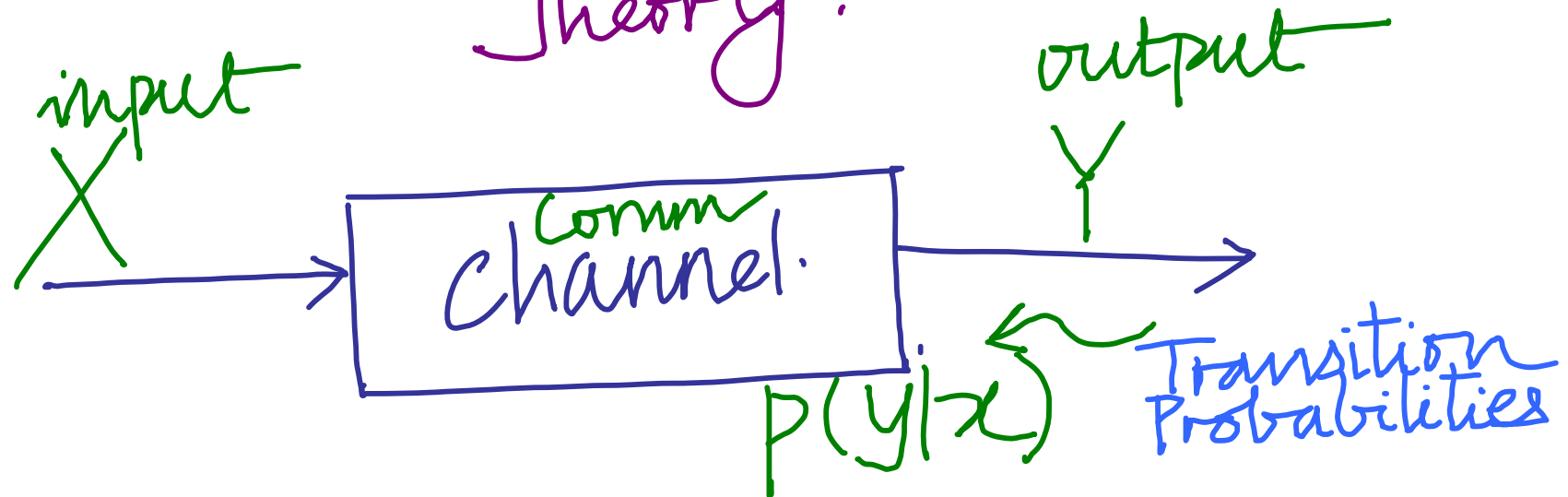


*

CHANNEL

CAPACITY

one of the
Fundamental
Results in information
Theory.



$$I(X;Y) = H(X) - H(X|Y) \\ = H(Y) - H(Y|X)$$

uncertainty
in X

uncertainty
in X on
observing Y

✓ uncertainty about
 X that is resolved on
observing Y .

Channel Capacity:

Denotes the maximum rate at which information can be transmitted over the channel.

$$C = \max_{Pr(X)} I(X; Y) \text{ bits/symbol!}$$

maximum of mutual information between input X & output Y .

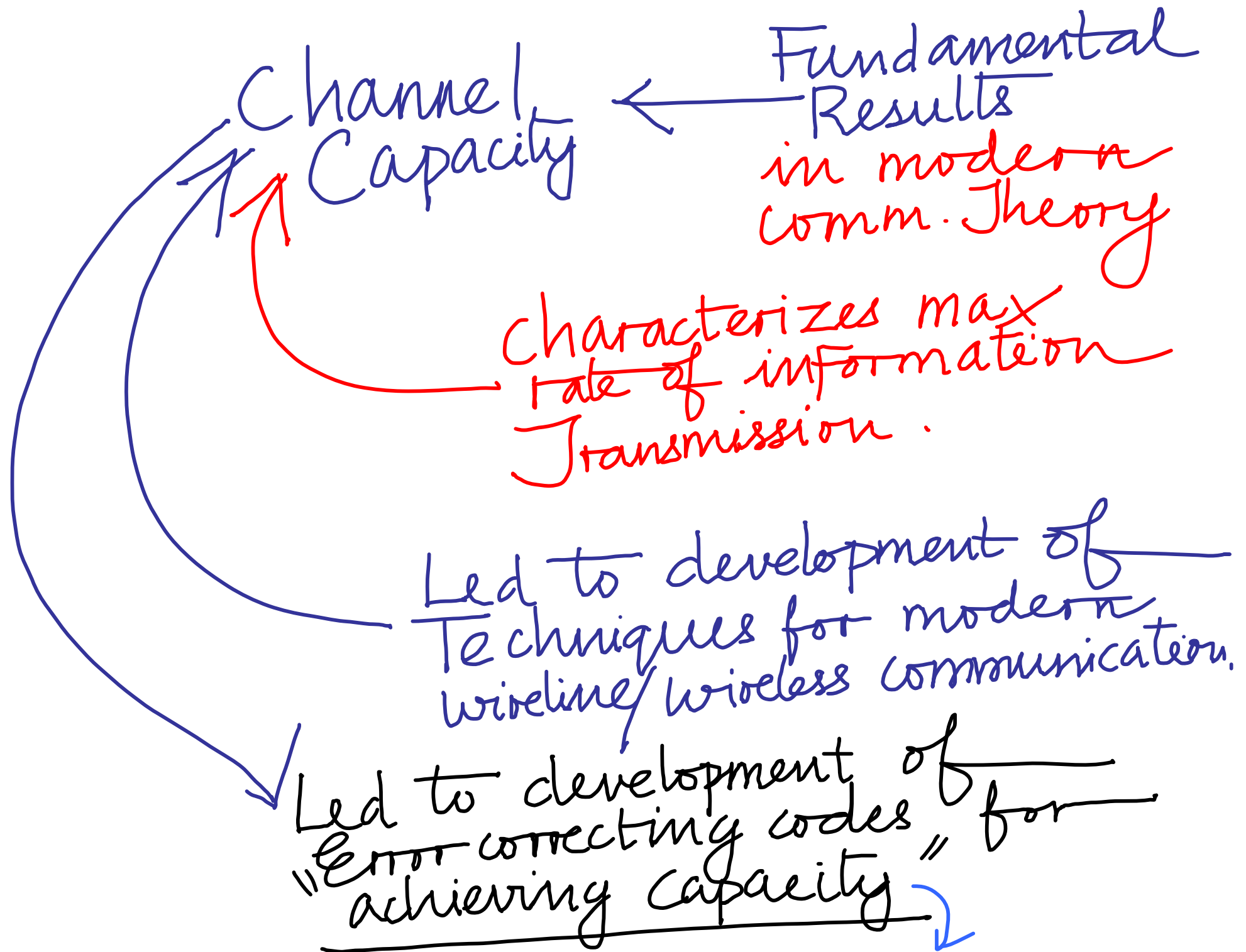
maximum over all input probability Distributions.

↓
Claude E. Shannon

"Father of Information Theory"

originally published in
Landmark research paper
titled,

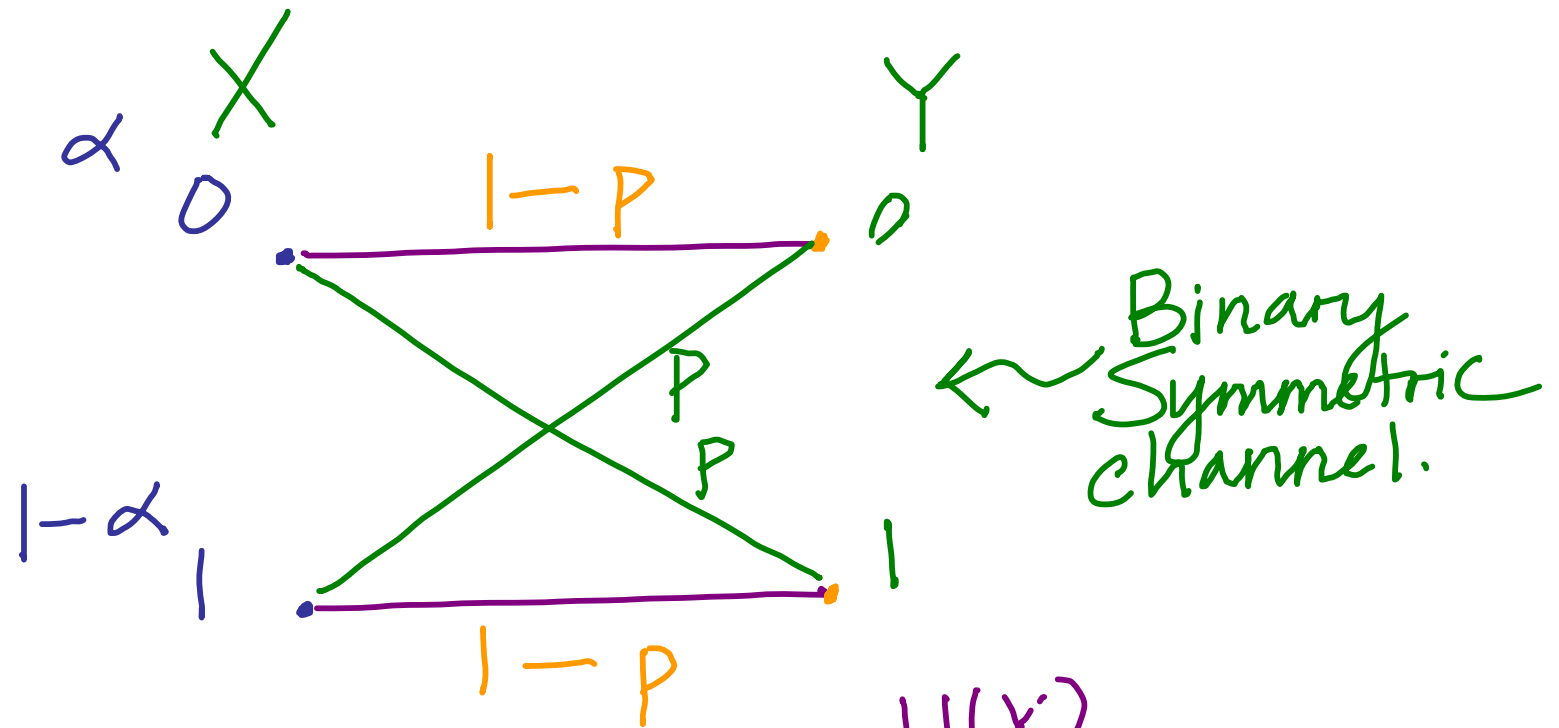
"A Mathematical Theory
of Communication".



Basically [←]Transmitting
information at rate
close to Capacity.

C — Rate at which
information can be
sent over channel with
arbitrarily low probability
of Error.

Example: $\swarrow$ BSC
Binary Symmetric Channel.



$$I(X; Y) = \overbrace{H(\alpha + p - 2\alpha p)}^{H(Y)} - \underbrace{H(p)}_{H(Y|X)} \quad \text{constant}$$

$H(p) = \text{constant}$
depends only on p .

$$C = \max_{\alpha} I(X; Y) = \left(\max_{\alpha} \frac{H(\alpha + p - 2\alpha p)}{H(p)} \right)$$

α completely characterizes input probability Distribution

max occurs for $\alpha + p - 2\alpha p = \frac{1}{2}$
Considering symmetry

$$\alpha = \frac{1}{2}$$
$$\alpha + p - 2\alpha p = \frac{1}{2} + p - 2 \cdot \frac{1}{2} p$$

$$= \frac{1}{2} + p - p$$

$$= \frac{1}{2}$$

$$H\left(\frac{1}{2}\right) = 1$$

$$= \max_{\alpha} H(\alpha + p - 2\alpha p).$$

$$C = \max_{\alpha} I(X; Y)$$

$$= \max_{\alpha} H(\alpha + p - 2\alpha p) - H(p).$$

$$= H\left(\frac{1}{2}\right) - H(p) = 1 - H(p)$$

$$C = 1 - H(p)$$

bits/channel use
For BSC

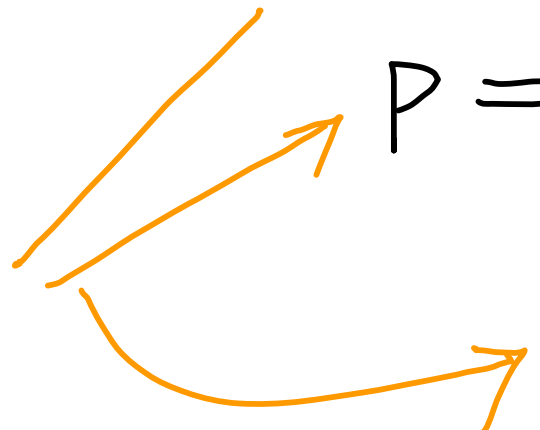
maximum
rate of information
transfer over BSC.

Capacity with $p \rightarrow 1-p$. $H(p) = H(1-p)$.

$$C = 1 - H(1-p)$$

$$= 1 - H(p).$$

$p=0$ $\downarrow$ $C = 1 - H(0)$
 $= 1 - 0 = 1$


$$P = 1 \quad C = 1 - H(1) \\ = 1 - 0 = 1$$

Capacity is maximum
for $P = 0$ or 1
 $= 1$ bit/channel
use.

$$P = \frac{1}{2} \\ C = 1 - H\left(\frac{1}{2}\right) \\ = 0 \text{ bits/symbol.}$$